EQUILIBRIUM AND STABILITY THEORY OF A POWER DISCHARGE IN A DENSE OPTICALLY TRANSPARENT PLASMA

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The equilibrium and stability of a power discharge in a dense optically transparent plasma is examined. It is shown that, contrary to the case of an optically nontransparent plasma, the temperature of a transparent discharge varies along the same characteristic scales as the other hydrodynamic quantities. Analysis of small oscillation spectra showed that such a discharge is unstable even within the framework of the geometrical optics approximation. The major portion of this paper is devoted to a study of the stability of a discharge with allowance only for bremsstrahlung; however, the conditions for the onset of instability are derived also for other types of radiation. Under the conditions studied, the general cause for the development of instabilities in a discharge in a transparent plasma is superheating that results from the inability of the weak emitted heat flux to compensate for the Joule heating of the plasma.

The utilization of optically nontransparent power discharges in dense plasmas as light sources for pumping lasers was discussed in [1]. In particular, the study made it possible to determine the discharge parameters for which a discharge stability sufficiently long for this purpose can be obtained for a required radiation intensity from the surface. In the present paper a theory is developed for an optically transparent plasma, in the case in which most of the radiation is transported by quanta with a mean free path exceeding the characteristic dimensions of the system.

The study of the equilibrium and stability of a transparent discharge is not solely of interest as such. It contributes to a better understanding of processes occurring at the boundary of a nontransparent discharge. Owing to the drop in particle density near the boundary of the nontransparent discharge, a transparent plasma layer is created for which the radiant (heat) transport approximation employed in [1] is no longer valid. The structure and stability of such a boundary can be analyzed on the basis of the results of this paper.

1. Formulation of the problem and the basic equations. Let us assume that the radiation flux from the discharge is sufficiently large to influence the latter both in the state of equilibrium and in the presence of oscillations. The corresponding conditions are given below. The complete system of magnetohydrodynamic equations for a plasma with allowance for radiation is written in the form [2, 3]

$$\begin{gathered}\operatorname{div}\mathbf{B}=0,\qquad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j}=\frac{4\pi}{c}\sigma\left\{\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right\},\\ -c\operatorname{rot}\mathbf{E}=\frac{\partial\mathbf{B}}{\partial t}=\operatorname{rot}[\mathbf{vB}]-\frac{c^2}{4\pi}\operatorname{rot}\left(\frac{1}{\sigma}\operatorname{rot}\mathbf{B}\right),\\ \rho\left[\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right]=-\nabla p+\eta\Delta\mathbf{v}+\\ +\left(\xi+\frac{\eta}{3}\right)\nabla(\nabla\mathbf{v})+\frac{1}{4\pi}[\operatorname{rot}\mathbf{B}\mathbf{B}],\\ \rho T\left[\frac{\partial s}{\partial t}+(\mathbf{v}\nabla)s\right]=\frac{j^2}{\sigma}+\sigma_{ij}'\frac{\partial v_i}{\partial x_j}+\operatorname{div}\chi\nabla T-\operatorname{div}S,\\ \frac{\partial\rho}{\partial t}+\operatorname{div}\rho v=0,\qquad p=p(\rho,T),\qquad s=s(\rho,T).\qquad(1.1)\end{gathered}$$

Here, ξ and η are viscosity coefficients, σ'_{ij} is the viscous-stress tensor, χ is the thermal conductivity coefficient, and S is the radiation flux vector.

At a temperature $T\sim 3$ to 10 eV, the plasma can be considered as a fully ionized ideal gas, and we use the following expression for the pressure p and entropy per unit mass s:

$$\begin{gathered}p=(1+z)N\varkappa T=\frac{(1+z)\varkappa T}{M}\rho,\\ s=-\frac{1+z}{M}\ln\rho+\frac{(1+z)c_V}{M}\ln\varkappa T+\text{const}.\qquad(1.2)\end{gathered}$$

Here, M is the ion mass, and z the effective ion charge. Under these conditions, the plasma conductivity is $\sigma=\alpha Z^{-1}T^{3/2}$, where $\alpha=4\cdot10^7$.

The system of equations (1.1) was written without considering the radiation energy (as compared to the thermal-particle energy). This is justified if

$$\delta\frac{\sigma^\circ T^4}{c}\ll p\approx N\varkappa T,\qquad(1.3)$$

where δ is a small quantity comparable in order to the ratio of the characteristic dimension of the plasma charge to the quantum mean free path, and $\sigma^0=5.67\cdot10^{-5}$ erg·cm^{-2}·degree^{-4}·sec^{-1} is the Stefan-Boltzman constant. Furthermore, in the following, the electron thermal conductivity is everywhere postulated small in comparison to the energy transfer by radiation, i.e.,

$$\operatorname{div}\chi\Delta T\ll\operatorname{div}S.\qquad(1.4)$$

In the following, these inequalities will be analyzed for the equilibrium obtained. At this point, we merely note that they are easy to fulfill, and that for temperatures $T>10^4$ °K, which are of interest when we use condition (1.4), inequality (1.3) is automatically satisfied at the same time. Finally, in the analysis of system (1.1), all effects associated with the viscous terms are neglected. For the equilibrium with $v_0=0$, examined below, this leads to a single requirement, namely that the oscillation frequencies must satisfy the inequalities

$$\omega\gg\frac{\xi k^2}{\rho},\quad\frac{\eta k^2}{\rho}.\qquad(1.5)$$

For an optically transparent plasma, we derive now an expression for the energy loss by radiation per unit volume, q_S. Considering the fact that for the overwhelming majority of directions of quantum propagation in a transparent medium, the radiation intensity is much smaller than the equilibrium intensity, we get

$$\begin{gathered}q_s=\operatorname{div}S=\int_0^\infty d\nu\int d\Omega\varkappa_\nu'(I_{\nu p}-I_\nu)\approx\int_0^\infty d\nu\int d\Omega\varkappa_\nu' I_{\nu p},\\ I_{\nu p}=\frac{2h\nu^3}{c^2}\left(\exp\frac{h\nu}{\varkappa T}-1\right)^{-1},\quad\varkappa_\nu'=\varkappa_\nu\left(1-\exp\frac{-h\nu}{\varkappa T}\right).\qquad(1.6)\end{gathered}$$

Here, $I_{\nu p}$ is the equilibrium radiation intensity, and $\varkappa_\nu'$ is the absorption coefficient with allowance for "re-emission."

The approximation made in (1.6) means that in the expansion of q_s in powers of $\delta \sim r_0/l_1$, only the zero-order term is retained. By evaluating integral (1.6) for the case of electron bremsstrahlung in an ion field, when

$$\varkappa_\nu = 4.1 \cdot 10^{-23} z^3 N^2 T^{-1/2} \left(\frac{h\nu}{\varkappa T}\right)^{-3},$$

we get

$$q_s = \gamma_0 \sqrt{T} N^2 Z^3 \qquad (\gamma_0 = 1.4 \cdot 10^{-27}). \tag{1.7}$$

In the general case [2], it is convenient to write q_s in terms of the quantum mean free path in the medium

$$q_s = \frac{4\sigma^\circ T^4}{l_1}, \qquad l_1(\rho, T) = \int_0^\infty I_{\nu p}\, d\nu \left(\int_0^\infty \varkappa_\nu' I_{\nu p}\, d\nu\right)^{-1}. \tag{1.8}$$

2. Equilibrium state of the charge. Before analyzing the stability of the discharge, let us examine the equilibrium problem. The energy balance in the discharge is ensured by Joule heating, on the one hand, and by volume radiation, on the other hand. From system (1.1) it can be readily seen that in the steady equilibrium state the field E_0 is uniform across the plasma (for $v_0 = 0$). The pressure, density, plasma temperature, current, and the magnetic field are functions of the coordinates. The spatial distribution of these quantities is defined by the equations* (the subscript zero refers to equilibrium values)

$$\begin{gathered}\mathrm{rot}\, \mathbf{B}_0 = \frac{4\pi}{c}\mathbf{j}_0 = \frac{4\pi}{c}\sigma_0 \mathbf{E}_0 = \frac{4\pi}{c}\frac{\alpha}{z} T_0^{3/2}\mathbf{E}_0, \\ \nabla p_0 = \frac{1}{4\pi}[\mathrm{rot}\, \mathbf{B}_0 \mathbf{B}_0], \qquad p_0 = \frac{(1+z)\varkappa}{M}\rho_0 T_0, \\ \sigma_0 E_0^2 = \gamma_0 \sqrt{T_0} N_0^2 z^3, \qquad \sigma_0 = \alpha z^{-1} T_0^{3/2}.\end{gathered} \tag{2.1}$$

From the equation of state and the energy balance equation, it follows that

$$p_0 = \left(\frac{\alpha E_0^2 \varkappa^2 (1+z)^2}{\gamma_0 z^4}\right)^{1/2} T_0^{3/2} \equiv \beta_0 T_0^{3/2}. \tag{2.2}$$

This result is independent of the discharge geometry. As in [1], the following analysis is performed for two types of discharge: a plane (surface) discharge, and a simple cylindrical discharge (z-pinch). Let us examine the plane discharge first. Eliminating B_0 and T_0 from the equilibrium equations (2.1), we get an equation for p_0,

$$\frac{\partial p_0}{\partial x} + \frac{\alpha_1 p_0}{\sqrt{2\pi\beta_0}}\sqrt{p_0(0) - p_0} = 0 \qquad \left(\alpha_1 = \frac{4\pi\alpha E_0}{cz}\right). \tag{2.3}$$

Here, $p_0(0)$ is the pressure at the discharge axis. From Eqs. (2.3), (2.2), and (2.1), we get the equilibrium values of p_0, T_0, and B_0 for the two-dimensional case,

$$\begin{gathered}p_0 = p_0(0)\frac{4e^{-\gamma x}}{(1+e^{-\gamma x})^2}, \qquad T_0 = T_0(0)\left(\frac{p_0}{p_0(0)}\right)^{2/3}, \\ B_0 = \sqrt{8\pi p_0(0)}\,\frac{1 - e^{-\alpha x}}{1 + e^{-\gamma x}} \qquad \left(\gamma = \frac{\alpha_1 \sqrt{p_0(0)}}{\sqrt{2\pi\beta_0}}\right),\end{gathered} \tag{2.4}$$

where γ is the characteristic dimension of a plane transparent discharge.

It is now easy to determine the radiation energy loss by a substance situated in a plane discharge, as referred to the unit area of the discharge, Q:

$$Q = \int_{-\infty}^{\infty} q_s\, dx = \frac{4\gamma_0\beta_0^2 z^3}{(1+z)^2\varkappa^2\gamma} T_0^{3/2}(0) = \frac{cE_0 B_0(\infty)}{2\pi}. \tag{2.5}$$

Although in the region $|x| \gg 1/\gamma$, the relations obtained do not hold because of the abrupt temperature drop and the formation of neutral particles, integration in formula (2.5) is extended to infinity, since the plasma density in this portion of the discharge also drops abruptly and the number of neutral particles is negligible in comparison to the total number of charged particles in the discharge. Finally, for the current, we have

$$I_0 = \int_{-\infty}^{\infty} j_0\, dx = c\left(\frac{2P_0(0)}{\pi}\right)^{1/2}. \tag{2.6}$$

Let us now examine the equilibrium state in the cylindrical case. By eliminating p_0 and T_0 from system (2.1) and introducing the variable $y = -1 + \alpha_1 B_0 r/4\pi\beta_0$, we get the equation

$$\left(ry' - y + \frac{1}{2}y^2\right)' = 0. \tag{2.7}$$

Solving this equation with the boundary condition $y = -1$ and $r = 0$ for hydrodynamic equilibrium values, we get the following expression

$$\begin{gathered}p_0 = \frac{p_0(0)}{(1 + r^2/r_0^2)^2}, \qquad T_0 = T_0(0)\left(\frac{p_0}{p_0(0)}\right)^{2/3}, \\ B_0 = \sqrt{8\pi p_0(0)}\,\frac{r/r_0}{1 + r^2/r_0^2},\end{gathered} \tag{2.8}$$

where $r_0 = 4\gamma^{-1}$ is the characteristic dimension of a transparent cylindrical discharge. The energy removed by radiation from a discharge of unit length is

$$Q = \int_0^\infty 2\pi r q_s(r)\, dr = \frac{16\gamma_0\beta_0^2 z^3}{(1+z)^2\varkappa^2\gamma^2} T_0^{3/2}(0). \tag{2.9}$$

The total current in the cylindrical case is

$$I_0 = \frac{2c^2\varkappa(1+z)}{\sqrt{\alpha\gamma_0 z}} \approx \frac{1+z}{z} 0.3 \cdot 10^6 a. \tag{2.10}$$

It can be seen that for a transparent cylindrical discharge with bremsstrahlung, for a given ionization level, the total current is a dc current that is independent of N_0, T_0, and E_0. A similar result has been obtained in [4] for a discharge in a high-temperature thermonuclear plasma.

On the basis of the equilibrium solutions obtained, it is easy to evaluate the plasma parameters for which the inequalities (1.3), (1.4) as well as the transparency condition $l_1 \gg 1/\gamma$ are fulfilled. It is found that the electron thermal conductivity at the discharge is so

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*The effective ion charge z is a function of the temperature. In the temperature range examined, when no neutral particles are contained in the plasma, this relationship is weak ($z \sim T^\beta$, where $\beta \lessdot 0.5$; with respect to magnitude, $z \approx 2$). This relationship will be neglected in the following.

small that it can be safely neglected everywhere. The corresponding condition has the form

$$T_0^{7/2}(0) \lesssim 10^{20} E_0, \qquad (2.11)$$

while the transparency condition for the discharge can be written as

$$T_0^{13/2}(0) \gtrsim 5 \cdot 10^{10} E_0. \qquad (2.12)$$

Finally, inequality (1.3) is fulfilled (assuming $\delta \sim 1$), if

$$T_0^{5/2} < 10^{16} E_0. \qquad (2.13)$$

As previously noted, at temperatures $T_0 > 10^4$ °K, inequality (2.11) is of a higher power than (2.13). On the other hand, inasmuch as only such temperatures are of interest, only inequalities (2.11) and (2.10) need be satisfied. For $E_0 \sim 0.1 \sim 1$ CGSE they can be readily satisfied in the range $2 \cdot 10^4 < T_0 < 5 \cdot 10^{5}$ °K.

It is noteworthy that, whereas inequality (1.4), in virtue of it being fulfilled at the discharge axis, is fulfilled everywhere, the inequality $\delta < 1$ (and, consequently, also (1.3)) no longer holds for $|x| > 1/\gamma$. This is associated with an abrupt drop in discharge temperature at these distances, owing to which the assumption that the dense plasma is fully ionized is no longer justified.

3. Stability of the discharge. We examine now the stability of the discharge with respect to the small perturbations

$$\rho \to \rho_0 + \rho_1, \; T \to T_0 + T_1,$$

$$p \to p_0 + p_1, \; \mathbf{B} \to \mathbf{B}_0 + \mathbf{B}_1, \; \mathbf{v}.$$

The perturbations are assumed to depend on time and on the coordinates as follows:

$$f_1 = f_1(x) \exp(-i\omega t + ik_y y + ik_z z),$$
$$f_1 = f_1(r) \exp(-i\omega t + im\varphi + ik_z z),$$

for a plane and a cylindrical discharge, respectively. A linearized system of equations, analogous to (1.1), was obtained in [1] and, hence, we do not repeat it here. It is enough to keep in mind that for a transparent plasma, the divergence of the perturbed radiation-flux vector, when bremsstrahlung alone is taken into account, is equal to

$$q_{s_1} = \operatorname{div} s_1 = g_{s0}\left(\frac{T_1}{2T_0} + 2\,\frac{\rho_1}{\rho_0}\right). \qquad (3.1)$$

In the general case, we have

$$q_{s1} = \frac{\partial q_{s0}}{\partial \rho_0}\rho_1 + \frac{\partial q_{s0}}{\partial T_0} T_1. \qquad (3.2)$$

As distinct from an optically dense discharge, instability of a transparent discharge occurs, as will be shown below, even in the geometrical optics approximation; in this case, the dispersion equations for plane and cylindrical discharges differ only by the trivial replacement $k_y \to m/r$ and, therefore, the analysis is limited to the plane discharge. Let us examine first the simple case in which $k_y = k_z = 0$ and radiation is pure bremsstrahlung. In this case, the system of linearized equations reduces to two equations for the quantities p_1 and $v = p_1 + (B_0 B_1/4\pi)$,

$$\frac{3}{2} p_1 + \frac{5}{2}\frac{v_{s0}^2}{\omega^2}\frac{\partial^2 v}{\partial x^2} +$$
$$+ \frac{5}{2}\frac{\partial v}{\partial x}\frac{1}{\omega^2}\frac{\partial v_{s0}^2}{\partial x} + \frac{1}{\omega^2}\frac{\partial v}{\partial x}\frac{B_0}{4\pi\rho_0}\frac{\partial B_0}{\partial x} +$$
$$+ \frac{ic^2}{4\pi\sigma_0\omega}\frac{\partial \ln B_0}{\partial x}\left[\frac{\partial}{\partial x}(p_1 - v) - (p_1 - v)\frac{\partial \ln B_0}{\partial x}\right] -$$
$$- \frac{p_1}{p_0}\frac{ic^2}{8\pi^2\sigma_0\omega}\left(\frac{\partial B_0}{\partial x}\right)^2 = 0,$$

$$v - p_1 + \frac{v_A^2}{\omega^2}\frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial x}\frac{B_0}{4\pi\omega^2}\frac{\partial}{\partial x}\frac{B_0}{\rho_0} + \frac{3ic^2}{32\pi^2\sigma_0\omega} \times$$
$$\times \frac{\partial B_0}{\partial x}\frac{\partial}{\partial x}\left[p_0^{-1}\left(p_1 + \frac{v_{s0}^2}{\omega^2}\frac{\partial^2 v}{\partial x^2}\right)\right] -$$
$$- \frac{B_0}{2}\frac{\partial}{\partial x}\left[\left(\frac{\partial B_0}{\partial x}\right)^{-1}\left(\frac{3}{2}p_1 + \frac{5}{2}\frac{v_{s0}^2}{\omega^2}\frac{\partial^2 v}{\partial x} + \right.\right.$$
$$+ \frac{5}{2}\frac{\partial v}{\partial x}\frac{1}{\omega^2}\frac{\partial v_{s0}^2}{\partial x} + \frac{v_A^2}{\omega^2}\frac{\partial \ln B_0}{\partial x}\frac{\partial v}{\partial x} +$$
$$\left.\left. + \frac{p_1}{p_0}\frac{ic^2}{8\pi^2\sigma_0\omega}\left(\frac{\partial B_0}{\partial x}\right)^2\right)\right] = 0,$$

$$v_{s0} = \sqrt{\frac{(1+z)\varkappa T_0}{M}}, \quad v_A = \frac{B_0}{\sqrt{4\pi\rho_0}}. \qquad (3.3)$$

Here, v_{s0} is the speed of isothermic sound in the plasma, and v_A the Alfvén velocity. We analyze system (3.3) in the geometrical optics approximation, i.e., for oscillations at a wavelenght smaller than the characteristic dimension of the plasma discontinuity

$$k_x \frac{1}{\gamma} \sim \frac{1}{\lambda_x \gamma} \gg 1. \qquad (3.4)$$

Here $\lambda_x \sim k_x^{-1}$ is the wavelength of the oscillations in the direction of a discontinuity. System (3.3) leads to the following eikonal equation [5]:

$$\omega^2\left(\omega^2 + \frac{i\omega c^2 k^2}{4\pi\sigma_0} - k^2 v_A^2\right) -$$
$$- k^2 v_s^2\left(\omega^2 + \frac{i\omega c^2 k^2}{4\pi\sigma_0}\right) + \frac{c^2 k^2 j_0^2}{6\pi\sigma_0^2 p_0}(\omega^2 - 3k^2 v_s^2) = 0. \qquad (3.5)$$

This equation can be readily solved with respect to ω in the two limiting cases $\omega \gg kv_s$ and $\omega \ll kv_s$, where it reduces to quadratic equations. The solution, then, can be written from a unified point of view

$$\omega_{1,2} = -\frac{ic^2k^2}{8\pi\sigma_0 t^2} \pm \frac{1}{2t^2}\left(-\frac{c^4k^4}{16\pi^2\sigma_0^2} - \frac{2c^2k^2t^2}{\alpha\pi\sigma_0^2 p_0}\right)^{1/2}, \qquad (3.6)$$

where

$$t = \begin{cases} 1 + v_A^2/v_s^2 \\ 1 \end{cases}, \quad \alpha = \begin{cases} 1 & \omega > kv_s \\ 3 & \omega < kv_s \end{cases}, \quad v_s^2 = \frac{5}{3} = v_{s0}^2.$$

It is noteworthy that in virtue of the condition for the applicability of the geometrical optics approximation (3.4), the first term under the radical sign in expression (3.6) is k/γ times greater than the second term. Therefore, the large root in (3.6),

$$\omega_2 = -i\,\frac{c^2k^2}{4\pi\sigma_0 t^2},$$

corresponds to the damped oscillations, and describes the penetration of the magnetic field into the plasma. It is this root that determines the transient period of the equilibrium state in the discharge (the time required for the electric field of the discharge to equalize across the plasma) $\tau \sim 1/\omega_2$. The small root, on the other hand, corresponds to aperiodic unsteady oscillations with a damping constant

$$Im\,\omega_1 \approx \frac{2j_0^2}{\sigma_0 p_0}\,\frac{t^2}{\alpha}\,. \tag{3.7}$$

The form of the damping constant clearly indicates that instability is associated with a finite conductivity of the plasma and is caused by ohmic heating. Instability is due to superheating, and is attributable to the fact that in an optically transparent plasma, the radiation emitted from the plasma is not capable of compensating for the increasing temperature fluctuations caused by Joule heating.

The high-frequency instability in the region $\omega > kv_s$ is not associated with the hydrodynamic motion of the plasma, and is caused solely by an increase in plasma temperature. Such an instability can occur only in a poorly conducting plasma with a sufficiently low temperature, where $\gamma c^2 > 4\pi\sigma_0 v_s$. With increasing plasma temperature, this inequality no longer holds, and the oscillations will stabilize; the instability liquidates itself. A high-frequency instability, induced by superheating, is therefore not dangerous. A low-frequency instability in the range $\omega < kv_s$ is more dangerous, since its development is accompanied by hydrodynamic motion of the plasma. Furthermore, such an instability can occur both in a poorly conducting low-temperature plasma and in a highly conducting high-temperature plasma*.

The danger of a low-frequency instability is enhanced by the circumstance that it can occur also at $k_z \neq 0$. Indeed, for $k_z \neq 0$, the dispersion equation of the oscillations in the geometrical optics approximation takes the form

$$\omega^2\left(\omega^2 + \frac{i\omega c^2k^2}{4\pi\sigma_0} - k^2 v_A{}^2\right) - k^2 v_s{}^2\left(\omega^2 + \frac{i\omega c^2 k^2}{4\pi\sigma_0}\right) + \frac{c^2k_x{}^2 j_0{}^2}{6\pi\sigma_0{}^2 p_0}\left[\left(1 - 2\,\frac{k_z{}^2}{k_x{}^2}\right)\omega^2 - 3k^2 v_s{}^2\right] = 0 \quad (k^2 = k_x{}^2 + k_z{}^2)\,. \tag{3.8}$$

In the range of low frequencies $\omega < kv_s$ the instability under study is conserved, although the increment of its development decreases by a value of k_x^2/k^2. However, in the range $\omega > kv_s$, at $k_z \neq 0$, the oscillations stabilize, provided $2k_z^2 > k_x^2$.

In view of the complexity of the system (1.1), a stability analysis for the case $k_y \neq 0$ was not performed. The authors are of the opinion, however, that allowance for other than zero values of k_y should not lead to a stabilization of low-frequency oscillations with $\omega > kv_s$, as is the case for an instability induced by superheating in a high-temperature plasma [6].

Let us now examine briefly the case of a radiation of general type in a transparent body, where expression (3.2) holds for q_{s1}. The dispersion equation for oscillations with $k_y = k_z = 0$, in the geometrical optics approximation, has the form

$$\omega^2\left(\omega^2 + \frac{ic^2k^2\omega}{4\pi\sigma_0} - k^2 v_A{}^2\right) - k^2 v_s{}^2\left(\omega^2 + \frac{i\omega c^2k^2}{4\pi\sigma_0}\right) + \frac{\omega c^2 k^2}{6\pi\sigma_0 p_0}\left(\frac{3}{2}\,\frac{j_0^2}{\sigma_0} - T_0\,\frac{\partial q_{s0}}{\partial T_0}\right) - \frac{c^2k^4}{6\pi\sigma_0\rho_0}\left(\frac{3}{2}\frac{j_0^2}{\sigma_0} + \rho_0\,\frac{\partial q_{s0}}{\partial \rho_0} - T_0\,\frac{\partial q_{s0}}{\partial T_0}\right) = 0\,. \tag{3.9}$$

From this equation, together with the known relationship $q_{s0}(\rho_0, T_0)$, it is possible to determine the instability boundary and the increment of instability development. It can be seen that radiation, generally speaking, can have both a stabilizing and a destabilizing effect on a plasma instability produced by superheating. Thus, in the high frequency range $\omega > kv_s$, where the plasma density may be considered constant throughout the oscillation process, radiation has a stabilizing effect, under the condition that

$$T_0\left(\frac{\partial q_{s0}}{\partial T_0}\right)_{\rho_0} > 0\,.$$

If, in addition, the inequality

$$T_0\left(\frac{\partial q_{s0}}{\partial T_0}\right)_{\rho_0} > \frac{3}{2}\,q_{s0}\,,$$

is fulfilled, radiation will fully stabilize this type of plasma instability.

In the opposite range, the plasma pressure remains constant during the oscillation process. Here, radiation has a stabilizing effect if

$$T_0\left(\frac{\partial q_{s0}}{\partial T_0}\right)_{p_0} = T_0\,\frac{\partial q_{s0}}{\partial T_0} - \rho_0\,\frac{\partial q_{s0}}{\partial \rho_0} > 0.$$

The instability induced by superheating is fully stabilized under the following condition:

$$T_0\left(\frac{dq_{s0}}{dT_0}\right)_{p_0} > \frac{3}{2}\,q_{s0}\,.$$

It should be noted that Eq. (3.9) is particularly useful in the analysis of oscillations in a discharge at distances of $|x| > 1/\gamma$ from the discharge axis, since, in this range, there appears a marked line spectrum which is caused by the formation of neutral atoms in the plasma radiation, and the condition of considering bremsstrahlung alone no longer holds.

4. Discussion of the results, and conclusions. When summing up the analysis of the equilibrium and stability of a power discharge in an optically transparent plasma, the instability of the discharge deserves to be mentioned first. As has been shown, the cause for this instability is the smallness of the energy flux removed by radiation as compared to ohmic heating. As distinct from an optically dense plasma, the temperature fluctuations in a transparent plasma cannot be fully dissipated by radiation, as a result of which they build up at an increment of $Im\,\omega \approx 1_0^2/\sigma_0 p_0$. By making use of the equilibrium obtained, the increment can be written in the form $Im\,\omega \approx 4\cdot 10^5 E_0$. From here it follows that for $E_0 \sim 0.1$–1 CGSE, instability develops during the time $\tau \sim 10^{-5}$ sec. This time is comparable with the hydrodynamic time r_0/v_s, and is smaller than the time normally required for sustaining a discharge when using it as a light source for pumping lasers. Therefore, it is inadvisable to use a completely transparent discharge as a light source.

In the case of a discharge in an optically dense plasma, the instability under consideration can develop also, but only in a narrow transparent layer about the nontransparent plasma which is responsible for the principal portion of the radiation. By adding the time required for instability to develop to the time required for the disturbances to travel from the transparent to the nontransparent layer, one arrives at the conclusion that development of instability at the periphery of the discharge when $r_p \gg r_0$ is unlikely to have any greater influence on the central nontransparent portion of the discharge.

The problem of the state which the plasma assumes as a result of the development of instabilities produced by superheating is of essential interest. A strict answer cannot be obtained in linear approximation, so that analysis of the nonlinear problem becomes necessary. In spite of that, simply judging from the maximum increment of instability development, it appears that instability should manifest itself by the formation of filaments or layers, characterized by a higher or a lower plasma conductivity, which extend in the direction of the total current in the discharge.

In conclusion, it should be noted that instability of a completely transparent discharge makes it less promising for laser pumping purposes. On the other hand, a totally nontransparent discharge (blackbody) may prove to be unprofitable in terms of energy, due to the substantial radiation energy losses in the far ultraviolet at photon energies of $h\nu \geq 3\varkappa T$. Of particular interest in this connection is the intermediate-

*The instability examined is analogous in nature to an instability induced by superheating in a high-temperature plasma in a strong longitudinal magnetic field which freezes the thermal conductivity of the plasma across the field [6].

type "semitransparent" discharge, where, in principle, it is possible to favorably combine stability with an adequately high efficiency.

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